APPLICATION OF MULTIFRACTAL ANALYSIS FOR IMAGE CLASSIFICATION AND DENOISING

Milorad Paskaš1, Irini Reljin2
1Inovation Center of School of Electrical Engineering, University of Belgrade
2 School of Electrical Engineering, University of Belgrade

Abstract: Fractal approach is successfully implemented in many applications within 2D signal processing. Fractal synthetic concept is applied for image compression with high compression ratios while fractal and multifractal descriptors provide classification of textural images with high accuracy. Various multifractal models are presented in literature. This paper summarizes them and illustrates new multifractal techniques applied for image classification and image denoising.

Keywords: Fractal dimension, multifractal spectra, image classification, denoising.

1. Introduction

Fractal model arose in 1960s for analysis of variables from econometrics in papers of Benoit Mandelbrot [1]. From the beginning fractal model presented a great potential in modeling nonlinear processes. However, fractality can be traced almost hundred years before, in papers of mathematicians who investigated non-differentiable functions. Non-differentiable functions, as it turned out, were seen as functions with non-integer derivative orders. This led to geometrical analysis of nowhere differentiable functions and generalization of their dimension to non-integer values. Therefore, functions (or more generally – sets) with non-integer dimensions are called fractals. Fractal dimensions are in general defined through measures and definition of a measure determines the type of a dimension. Namely, this leads to fact that two different fractal measures give different dimension values for the same object (signal). Thus, construction of the new fractal measures should provide characterization of particular features within the signal where each measure (and respecting dimension) should describe a specific quality of the signal.

In the light of dimensionality there can be seen multifractality introduced later by the same author for description of fractals with non-uniform dimensions. In other words, multifractal is a union of fractals with various dimension values. This notion opened a place for a new term – monofractality, which considers fractals with uniform dimensionality.

Description of multifractality is more difficult task than description of monofractality. It is needed to locate pixels with the same dimension value; this is done using local multifractal measure that gives local dimension or Hölder exponent. Pixels with the same dimension within the signal form the monofractal subset and the so-called
global dimension is calculated for each monofractal subset. Global dimensions dependence of corresponding local dimensions is called multifractal spectrum or singularity spectrum [2]. The shape of the curve representing this dependence is concave and smooth with self-similar measures. In general, multifractal spectrum is not smooth and variations of spectrum reveal some characteristics of the signal. Local dimension matrices illustrate local behavior of the signal or the trend of the signal in the pixel’s neighborhood. Global dimensions demonstrate a global trend of the whole regions within the signal. Therefore, complete mapping from local to global dimension embodied in multifractal spectrum gives a ‘signature’ of a signal. However, the multifractal signature is not unique feature of the signal and there are other parameters defined that supplement spectrum.

Its greatest development multifractality achieved in physics for description of turbulent flows and sensitivity of the system to initial conditions [3][4]. That is the point where the fractal model came into contact to the chaos theory. This is the reason why interpretations of many fractal parameters, like Legendre multifractal spectrum or partition function, came from physics (thermodynamics). In 1980s fractal model was implemented for 1D and 2D signal processing. Fractal techniques from that period are still dominant in signal processing. Those techniques are based on geometrical interpretation of an image where the image is defined as a geometrical surface or volume. This implicitly leads to definition of the pixel intensity as a (third) spatial coordinate that is not a common notion in signal processing community. Multifractal techniques that consider pixel intensity as a function defined over spatial coordinates originate from 1990s. Although these two approaches are based on different models of the image, they are both still in use. These two approaches will be further explained in the next section.

Fractal model is applied in many areas of image processing. One of the most successful applications of the fractal model is in classification and segmentation of images [5], [6]. There are many algorithms of this model in medical image processing such as: radiograph images [7], mammograms, cytological images [8], forensics [9]. All listed applications rely on analytical fractal approach: the signal is assumed fractal or multifractal and parameters of fractal model are derived without information of the image formation. Image generation as a non-differentiable (fractal) function of two variables is involved in image compression where fractal model achieved outstanding results [10].

The focus of this paper is a presentation of novel fractal techniques for image classification, segmentation and adaptive denoising. The paper is organized as follows. The section 2 describes fractal descriptors from literature. The most of fractal techniques are divided into two classes depending on image perception; image can be modeled either as a geometrical object (surface, volume) or as a function of two (spatial) variables. Section 3 introduces fractal methods for image classification and segmentation. Proposed methods represent generalization of blanket method from the literature [11]. Blanket method is a monofractal technique while its multifractal adaptation is recommended in the geometrical and non-geometrical sense. Multifractal spectra are used as descriptors of images and pixels in the scope of classification. Section 4 illustrates fractal model application for adaptive image filtering. Local dimension matrices obtained by two multifractal measures are used as masks for low-pass filters from the literature. The requirements of the desired filter are the elimination of high-frequency noise and preservation of edges and textural regions. Section 5 gives conclusions of presented multifractal methods and directs to their improvements in the future.
2. Multifractal descriptors

The most frequently used multifractal (MF) features are *fractal dimension* and *multifractal spectrum* (MFS). Calculation of fractal dimension implies monofractal (unidimensional) model of the image, while multifractal spectrum implies multifractal modeling of the image. Apart from the classification of fractal methods on monofractal and multifractal ones, they can be categorized with respect to applied model of the image. Commonly used model of an image assumes pixel intensity as a function over two spatial coordinates. This model corresponds to a physical formation of an image and is considered as *non-geometrical*. Fractal dimension (or MFS) calculated under this model is, actually, a dimension of a function which defines pixel intensity mapping over spatial coordinates. The other image model most often used in fractal image processing interprets pixel intensity as a third spatial component. Therefore an image is represented by the surface embedded in a 3D space. The calculated fractal dimension is indeed a dimension of a geometrical surface defined by the image and this model is further considered as a *geometrical* model. This model is not based on the principles of physical formation of an image.

Geometrical methods are based on calculation of area or volume of the image surface. The only difference among those techniques is the procedure of calculating area and volume. Techniques from literature with geometrical modeling of the image are further summarized:

- **Differential box-counting (DBC) method** [12] is a modification of traditional box-counting method. This method implies covering of the image with 3D boxes, where pixel intensities are interpreted as a third spatial coordinate. The number of pixels included within the boxes at each scale is the measure. DBC method is one of the most frequently used techniques since of its simplicity. It is used for segmentation of images [13], classification of images [14], and as a descriptor of medical images [15]. There are many modifications of this method in the literature.

- **Blanket method** [11] proposes construction of upper and lower blankets based on minimum and maximum operators. The upper and lower blankets enclose the image surface and define its volume. The area of the image surface is then calculated as a ratio of volume and thickness of the volume. The process of calculation of blankets is iterative. Blanket method is successively implemented in segmentation of images [16], [17], as a feature vector in mammography [18], and in forensics [9].

- **Triangular prism surface area method** [19] interprets image surface as a superposition of triangular prisms surfaces over the uniform boxes. Triangular prism method is used for the analysis of satellite images [20].

- **Method based on mathematical morphology** [21], derives the area of the image surface indirectly over the volume as with the blanket method. Volume is defined by the image surface (from the bottom) and the dilated replica of the image (from the top). Area of the image surface is obtained by dividing the volume of the image by its thickness.
- **Variation method** [22] determines image dimension using volume, not the surface, as a fractal measure. The thickness of the volume for each pixel in the image is determined as a maximum difference between pixel intensities in the neighborhood defined by the scale.

Non-geometrical methods from the literature are diverse regarding interpretation of measure, although all of them interpret pixel intensity as a measure of a single pixel. They are further summarized:

- **Myopic capacities** [23] calculate local multifractal measure over pixels intensities within the scaled neighborhood. There are many different measures from this group and all of them interpret pixel intensity as a local measure of a single pixel. Myopic capacities are used for frame change detection [24], image segmentation [7], detection of objects in the infrared images [25].

- **Difference-based capacities** [26] are based on myopic capacities. They are based on difference of pixel intensities within the scaled neighborhood of each pixel.

- **Spatial derivative-based capacities** [27] involve spatial derivatives (gradient, Laplacian) into determination of local multifractal measures. Spatial derivatives are employed in interaction with Gaussian as with the edge detectors.

- **Implementation of integral transformations** [28] is frequently used approach for calculation of Hölder exponents. Fourier Transform (FT) or, more often, Wavelet Transform (WT) is used. In applications of WT the local dimensions (Hölder exponents) are calculated using vanishing moments property of specific wavelets. WT is applied in image denoising applications [29] and for detection of microcalcifications and masses in the mammograms [30], [31].

- **Fractional Brownian motion (fBm)** [2] regards a signal as a process with the variance that depends on time in a power-low manner. The calculation of local dimensions is accomplished using FT. FT implementation in MFA assumes calculation of power spectral density. Modeling images with fBm is one of the oldest techniques and it has been used in: textural image synthesis [32], classification and segmentation of images [33], and medical image analysis [34], [35].

3. Image classification using MF descriptors

Classification of textural images represents an issue to traditional classification techniques using features such as histograms. Multifractal spectrum represents one of the features that overcome this issue. In [36], [37] three multifractal methods based on blanket method are introduced. Blanket method is generalized for multifractal case. All three methods are designed in a way that overcomes the iterative construction of the multifractal measure. Thus, in order to calculate local dimensions on desired scale, all the smaller scales are not considered. Calculation of the local dimensions in Method 1 is a direct generalization of the original method according to multifractal techniques with
geometric approach. Upper and lower blankets are calculated in a manner where the pixel intensity is treated as a third spatial coordinate. Global dimensions are calculated using box-counting method. Final MFS obtained using this method is shown in Figure 1.a.

Method 2 brings non-geometrical approach where the upper and lower blankets are constructed without the (spatial) scale information. The monotony of those blankets is achieved with constant parameters for each blanket and they are mutually independent. Discrimination capacity of MFS provided by Method 2 is not influenced significantly by the choice of constant parameters. Spectra are only slightly shifted to the left and more narrow as the parameters increase. Global dimensions are calculated using box-counting method and the MFSs for three textures are shown in Figure 1.b.

Local dimensions in Method 3 are calculated using multifractal measure provided in Method 1. Thus, Method 3 can be seen as a geometrical method for calculation of local fractal dimensions. Global dimensions are calculated using original (monofractal) blanket method in such a way that global dimensions of each monofractal subset within the image are calculated using original blanket method. Here, embedded dimension is three and it is also the maximum global dimension. Figure 1.c. illustrates MFSs for three textures. In comparison to Figure 1.a. and Figure 1.b. it is obvious that maximum global dimensions in Methods 1 and 2 are 2. This is the consequence of gauge used in Method 3 which is of dimension 3.

In all three methods local dimensions are calculated using the lowest scales, as it is confirmed in conducted experiments where resulting MFSs are the most discriminative for the smallest scales. Scales considered in calculation of global dimensions are taken from the wide range – from the smallest to the largest (maximum scale is the half of the image size in pixels). Omission of the smallest scales leads to rounding shapes of MFS which are not discriminative anymore.

Classification results obtained by all three proposed methods exceed results obtained by the original method. Experiments are conducted on Brodatz database consisted of 110 textures of size 640x640 pixels. Each image from the database is divided

![Figure 1](image1.png)

**Figure 1 Illustration of MFS for three textures obtained by:**
- a) Method 1, b) Method 2, and c) Method 3.
into 25 images, thus each class contains 25 textures. Classification of textures from the database is executed using k-means algorithm. Classification is repeated for 400 times since k-means algorithm is sensitive to the initial conditions. Percentage of correctly classified textures using original method is 84.53%. Proposed multifractal methods give classification rates of: 93.78%, 92.42%, and 88.50%, respectively. Explanation of improved classification performances of the proposed methods is found in the size of descriptors used in the classification: original blanket method provides a single value (fractal dimension) as a descriptor, while multifractal methods give feature vectors of variable size. All the four experiments are carried under the same conditions. Further, average computation times are comparable for Methods 1 and 2, while Method 3 requires much higher computation time. This is due to iterative calculation of global dimensions in Method 3.

Classification of textural images can be exploited for the image segmentation. For that purpose MFS is used as a descriptor of each pixel. In other words, the MFS is calculated on a sliding window of a fixed size. Obtained spectrum is dedicated to a central pixel as its feature, and it is used for classifications of pixels or image segmentation. At first, local dimensions are determined on the whole image using Method 1. Box-counting method is used for calculation of global dimensions. Results of segmentation of collages of textures are illustrated in Figure 2 in case of two and four textures within the collage. As it is seen in Figure 2, the most erroneous segmentation results are zones containing the border of textures. The explanation of such pixel misclassification lies in the fact that MFS is calculated on the scaling neighborhoods and boundary pixels have neighborhoods consisted of pixels from different textural regions.

Figure 2 Segmentation of collages of textural images: a) collage consisted of two textures, b) segmentation result for image from a), c) collage consisted of four textures, d) segmentation results for image from c).
4. Adaptive image denoising using local MF features

Image denoising techniques suffer from blurring edges within the image. This is a result of low-pass (LP) filters used for noise reduction. Additive noise is assumed to have high-frequency components while some components of the signal also occupy high ranges in frequency domain. Edges and textural regions are such signal components. The image is usually presumed as a union of homogeneous regions where the pixel intensity within each region has Gaussian distribution with particular mean value and standard deviation. Within this model, the edges are considered as borders between two homogeneous regions. The transition of pixel intensity on the borders reflects in high frequencies in the frequency domain. Textural regions, on the other hand, are seen as the regions with a high degree of edges.

![Local dimension matrices are used as masks for adaptive filtering: a) original image, b) gradient image, c) local dimension image obtained with MAX measure, d) local dimension image obtained with MIN measure.](image)

Adaptive filters proposed in [37], [38] are based on local dimensions or Hölder exponents. Local dimensions provide information on local behavior of a multifractal measure and pixels with the same local dimension values (iso-Hölder regions) indicate unidimensional regions. Depending on multifractal measure definition there are corresponding local dimension matrices with different features accentuated within image. Two multifractal measures are used as edge-detectors in literature [39] – MAX and MIN measure – that is the obtained local dimension matrices extract edges. Measures MAX and MIN analyze trend of behavior of each pixel in its neighborhoods where MAX
measure calculates maximum pixel intensity and MIN measure calculates minimum pixel intensity in the various scales of the neighborhood. In other words, these two multifractal measures analyze maximum and minimum pixel intensities through different scales. Multifractal model suggests power-law dependence between measure and scale. In log-log plot it is reflected as a linear dependence.

Figure 4 Adaptive image filtering using local dimension obtained with MAX measure: a) original image and filtered images obtained by: b) averaging, c) Gaussian filter; d) adaptive filter with Gaussian filter. The size of filter window in all cases is 7 pixels.
Proposed adaptive denoising employs local dimension matrices obtained with MAX and MIN measures. Local dimension images (matrices) are in both cases analogue of the gradient image, where edges have high intensities and homogeneous regions are with low intensities. This is illustrated in Figure 3. In adaptive filtering scenario the local dimension matrices are used as masks for LP filters. In this way LP filter is favored in pixels with low local dimensions and LP filter is attenuated with pixels with high local dimensions, since high dimensions indicate edges. There are two LP filters employed in the analysis common in image denoising: averaging and Gaussian LP filter. Adaptive image denosing is shown in Figure 4 for both multifractal measures.

From Figure 4 effect of denoising and edge preservation using adaptive approach is evident. Non-adaptive approaches (averaging and Gaussian filter) treat all pixels within the image in the same manner and, as a result, the edges (and textures) are blurred. This is not the case with adaptive filter realizations. Besides the qualitative tests, quantitative measurements are studied. Effective average gradient (EAG) [40] values confirm improvements retrieved by visual inspection. In all the test images (natural and aerial images) EAG values approve better edge preservation in adaptive than in non-adaptive filtering. Further, EAG values favor adaptive filtering especially in the case of larger filtering windows.

5. Conclusion

This paper briefly represented fractal techniques for image processing from the literature. Novel multifractal methods for image classification, segmentation and adaptive filtering are represented. Classification methods use multifractal spectra as a descriptor of the image for classification. Represented segmentation scenario engages multifractal spectrum as a descriptor of a pixel and reduces segmentation to classification of pixels. Proposed adaptive image filters employ local dimension matrix as a mask for labeling edge-like regions in the image. Pixels in the regions with the high content of edges are minimally affected by low-pass filter, while the pixels in the homogeneous regions are filtered stronger and the noise, assumed to be of a high frequency, is eliminated. As a result, edges and textures in the filtered image are preserved.

References


Sadržaj: Fraktalni pristup se uspešno koristi u mnogim primenama u okviru obrade 2D signala. Fraktalni sintetički koncept se primenjuje za kompresiju slika sa visokim stepenom kompresije dok fraktalni i multifraktalni deskriptori obezbeđuju klasifikaciju teksturalnih slika sa visokom tačnošću. U literaturi su dati razni multifraktalni modeli. Ovaj rad daje njihov pregled i donosi nove multifraktalne postupke koji su primenjeni za klasifikaciju slika i eliminaciju šuma u slikama.

Ključne reči: Fraktalna dimenzija, multifraktalni spektar, klasifikacija slika, eliminacija šuma.

KORIŠĆENJE MULTIFRAKTALNOG PRISTUPA ZA KLASIFIKACIJU SLIKA I ELIMINACIJU ŠUMA

Milorad Paskaš, Irini Reljin